112 學年度科技校院四年制與專科學校二年制

統一入學測驗公告答案

考科代碼：4-00-MB

類　　別：共同科目

考　　科：數學(B)

題號	答案	題號	答案	題號	答案	題號	答案	題號	答案	題號	答案
1	A	11	C	21	C	31		41		51	
2	B	12	C	22	A	32		42		52	
3	D	13	B	23	B	33		43		53	
4	D	14	A	24	D	34		44		54	
5	A	15	B	25	D	35		45		55	
6	D	16	C	26		36		46		56	
7	C	17	C	27		37		47		57	
8	D	18	C	28		38		48		58	
9	B	19	A	29		39		49		59	
10	A	20	B	30		40		50		60	

1. 向量$(2,1)$

(A)$(-1,\frac{1}{2})$ $\Rightarrow\begin{cases}\dfrac{2}{-1}\neq\dfrac{1}{\frac{1}{2}}\text{ 不平行}\\ 2(-1)+1\cdot\dfrac{1}{2}\neq 0\text{ 不垂直}\end{cases}$

(B)$(1,\frac{1}{2})$ $\Rightarrow\begin{cases}\dfrac{2}{1}=\dfrac{1}{\frac{1}{2}}\text{ 平行}\\ 2\cdot 1+1\cdot\dfrac{1}{2}\neq 0\text{ 不垂直}\end{cases}$

(C)$(-\frac{1}{2},1)$ $\Rightarrow\begin{cases}\dfrac{2}{-\frac{1}{2}}\neq\dfrac{1}{1}\text{ 不平行}\\ 2(-\dfrac{1}{2})+1\cdot 1=0\text{ 垂直}\end{cases}$

(D)$(-1,-\frac{1}{2})$ $\Rightarrow\begin{cases}\dfrac{2}{-1}=\dfrac{1}{-\frac{1}{2}}\text{ 平行}\\ 2(-1)+1(-\dfrac{1}{2})\neq 0\text{ 不垂直}\end{cases}$

2. $f(x)=(2x^2-3)^5+3(x-1)^2$

各項係數的總和$=f(1)=(2-3)^5+3(1-1)^2=-1$

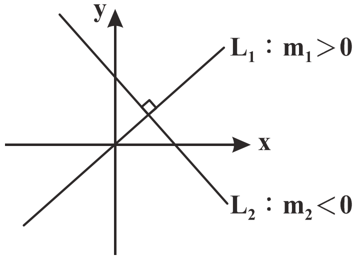
3. $3x^2-kx+h=0$

沒有實根 $\Rightarrow b^2-4ac<0$ $\Rightarrow(-k)^2-4\cdot 3\cdot h<0$ $\Rightarrow k^2-12h<0$

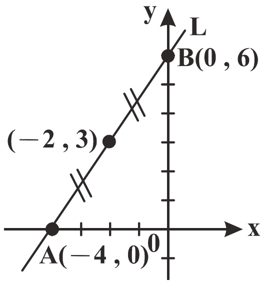
(A)$(-4,1)=(-4)^2-12\cdot 1=4$ (B)$(12,12)=12^2-12\cdot 12=0$

(C)$(5,2)=5^2-12\cdot 2=1$ (D)$(10,9)=10^2-12\cdot 9=-8<0$

4. 點(m_1 , m_2)落在第四象限



5. 斜率$=\dfrac{0-6}{-4-0}=\dfrac{3}{2}$



$L：y-0=\dfrac{3}{2}(x-(-4))$

$\Rightarrow f(x)=\dfrac{3}{2}(x+4)$

$\Rightarrow f(2)=\dfrac{3}{2}(2+4)=9$

$a+b+f(2)=-4+6+9=11$

6. x 誤看成 y，y 誤看成 x $\Rightarrow$ 圓心(1 , 2)，故原本題目之圓心應為(2 , 1)

(B)$x^2+y^2-4x-2y-11=0$

$\Rightarrow$ 圓心(2 , 1)，$r=\dfrac{1}{2}\sqrt{(-4)^2+(-2)^2-4(-11)}=4$，直徑$=8$

(D)$x^2+y^2-4x-2y+1=0$

$\Rightarrow$ 圓心(2 , 1)，$r=\dfrac{1}{2}\sqrt{(-4)^2+(-2)^2-4\cdot 1}=2$，直徑$=4$

7. (1)$(\vec{u}+\vec{w})\cdot\vec{u}=|\vec{u}+\vec{w}||\vec{u}|\cdot\cos75^\circ$

$\Rightarrow|\vec{u}|^2+\vec{u}\cdot\vec{w}=|\vec{u}+\vec{w}|\cdot 2\cdot\cos75^\circ$

$\Rightarrow 4+\vec{u}\cdot\vec{w}=|\vec{u}+\vec{w}|\cdot 2\cdot\cos75^\circ$

兩邊乘(−1)

(2)$(-\vec{u}-\vec{w})\cdot\vec{w}=|\vec{u}+\vec{w}||\vec{w}|\cdot\cos\theta$；(令$(-\vec{u}-\vec{w})$與$\vec{w}$的夾角為$\theta$)

$$\left.\begin{aligned}&\Rightarrow-\vec{u}\cdot\vec{w}-|\vec{w}|^2=|\vec{u}+\vec{w}|\cdot 2\cdot\cos\theta\\&\Rightarrow-4-\vec{u}\cdot\vec{w}=|\vec{u}+\vec{w}|\cdot 2(-\cos75^\circ)\end{aligned}\right\}\cos\theta=-\cos75^\circ\quad\therefore\theta=105^\circ$$

8. 原式 $\Rightarrow(\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\dfrac{1}{2^4}+\dfrac{1}{2^5})+(\dfrac{1}{2^2}+\dfrac{1}{2^3}+\dfrac{1}{2^4}+\dfrac{1}{2^5}+\dfrac{1}{2^6})x>0$

兩邊同乘 2^6 $\Rightarrow(2^5+2^4+2^3+2^2+2)+(2^4+2^3+2^2+2+1)x>0$

$\Rightarrow 31x>-62$ $\Rightarrow x>-2$

9. $m_L = -\dfrac{a}{4} = \dfrac{1}{2} \quad \Rightarrow a = -2$

L：$-2x + 4y + k = 0$

$d(O, L) = \dfrac{|k|}{\sqrt{(-2)^2 + 4^2}} = \sqrt{5}$

$\Rightarrow |k| = \sqrt{5} \cdot \sqrt{20} = 10 \quad \Rightarrow k = \pm 10$(負不合)

$\therefore a + k = -2 + 10 = 8$

10. 令 $f(x) = (x+2)(x-7)Q(x) + (ax+3)$

$(x-7)$為 $f(x)$的因式 $\quad \Rightarrow f(7) = 0 \quad \Rightarrow 7a + 3 = 0 \quad \Rightarrow a = -\dfrac{3}{7}$

$f(x) = (x+2)(x-7)Q(x) + (-\dfrac{3}{7}x + 3)$

$f(-2) = 0 + (-\dfrac{3}{7}(-2) + 3) = \dfrac{27}{7}$

11. 開口向下 $\quad \Rightarrow x^2$ 項係數$<0 \quad \Rightarrow n^2 - n - 12 < 0 \quad \Rightarrow (n-4)(n+3) < 0$

$\Rightarrow -3 < n < 4 \quad \Rightarrow$ 整數 $n = -2, -1, 0, 1, 2, 3$(有 6 個)

12. (1) $f(x) = g(x) \quad \Rightarrow (2a-b)x^2 + ax - 1 = 3x^2 + x - 1$

$\Rightarrow \begin{cases} 2a - b = 3 \\ a = 1 \text{代入上式得} b = -1 \end{cases} \quad \Rightarrow Q(x) = x - 1$

(2) $\dfrac{x}{Q(x)} + \dfrac{5}{x-2} = \dfrac{-1}{(x-2)Q(x)} \quad \Rightarrow \dfrac{x}{x-1} + \dfrac{5}{x-2} = \dfrac{-1}{(x-2)(x-1)}$

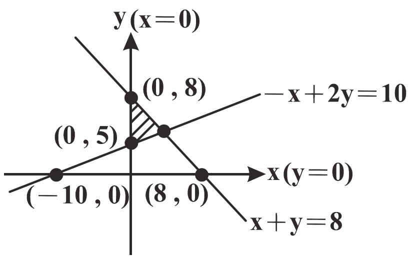
$\Rightarrow \dfrac{x(x-2)+5(x-1)}{(x-1)(x-2)} = \dfrac{-1}{(x-2)(x-1)} \quad \Rightarrow x(x-2) + 5(x-1) = -1$

$\Rightarrow x^2 + 3x - 4 = 0 \quad \Rightarrow (x+4)(x-1) = 0$

$\Rightarrow x = -4 = c \quad$ or $\quad x = 1$(不合，$\because$分母$\neq 0$)

(3) $a^2 + b^2 + c^2 = 1^2 + (-1)^2 + (-4)^2 = 18$

13.



14. $\cos 39^\circ \tan 39^\circ + \sin 30^\circ \tan 45^\circ \cos 60^\circ + \sin 129^\circ \tan 141^\circ$

$= \cos 39^\circ \cdot \tan 39^\circ + \dfrac{1}{2} \cdot 1 \cdot \dfrac{1}{2} + \sin(90^\circ + 39^\circ) \cdot \tan(180^\circ - 39^\circ)$

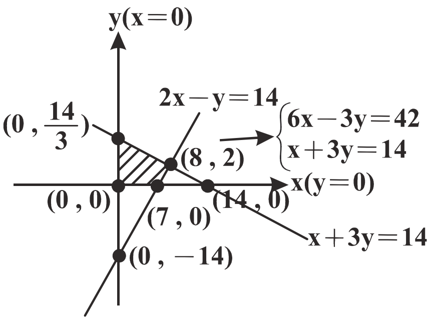
$= \cos 39^\circ \cdot \tan 39^\circ + \dfrac{1}{4} + (\cos 39^\circ)(-\tan 39^\circ) = \dfrac{1}{4}$

15. (1)第一週 $a_1=8$
第二週 $a_2=8+3$
第三週 $a_3=8+3+3$

$\left.\begin{array}{l}\text{第一週 } a_1=8\\ \text{第二週 } a_2=8+3\\ \text{第三週 } a_3=8+3+3\end{array}\right\}a_n=a_1+(n-1)d=8+(n-1)3=3n+5$

(2)$a_n>42.195\quad\Rightarrow 3n+5>42.195\quad\Rightarrow 3n>37.195\quad\Rightarrow n>12.\ldots\quad\Rightarrow$ 最小 $n=13$

16. $\begin{cases}(x,y)\cdot(1,3)=x+3y\leqq 14\\ (x,y)\cdot(2,-1)=2x-y\leqq 14\\ x\geqq 0\\ y\geqq 0\end{cases}$



$(x,y)\cdot(1,1)=x+y$ 的最大值？

$f(x,y)=x+y$

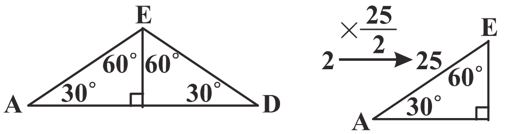
$f(0,0)=0$

$f(7,0)=7$

$f(8,2)=10$……最大

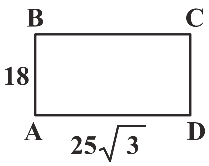
$f(0,\frac{14}{3})=\frac{14}{3}$

17. (1)



$\sqrt{3}\xrightarrow{\times\frac{25}{2}}\frac{25\sqrt{3}}{2}\quad\Rightarrow\overline{AD}=25\sqrt{3}$

(2)



長方形面積$=18\times 25\sqrt{3}=450\sqrt{3}$

18. $37\cdot\log_{10}x+31\geqq 70$

$\Rightarrow 37\cdot\log_{10}x\geqq 39$

(C)$x=12\quad\Rightarrow 37\log_{10}12=37(\log_{10}2^2+\log_{10}3)=37(0.6020+0.4771)=39.9267$

19. $\left.\begin{array}{l}\text{小楓}P=\dfrac{50+40}{100+200}=\dfrac{90}{300}\\ \text{小道}P=\dfrac{90+15}{200+100}=\dfrac{105}{300}\end{array}\right\}$小道比較高

20. (A)$(1400,1900)\quad\Rightarrow\begin{cases}1400\times 100+1900\times 200=520000\\ 1400\times 200+1900\times 100=470000\end{cases}$

(B)$(1600,1700)\quad\Rightarrow\left\{\begin{array}{l}1600\times 100+1700\times 200=500000\\ 1600\times 200+1700\times 100=490000\end{array}\right\rangle$不會超過預算

21. 連續三格的答案　出現 BAD $\left.\begin{array}{l} \underline{21\quad 22\quad 23}\quad 24\quad 25 \\ \quad\ \ \underline{22\quad 23\quad 24} \\ \quad\quad\quad\ \ \underline{23\quad 24\quad 25} \\ \text{組數}=3 \end{array}\right\}$

其中 1 組：$\underset{\text{B}}{\underline{21}}\quad\underset{\text{A}}{\underline{22}}\quad\underset{\text{D}}{\underline{23}}\quad\underset{\text{選 1}}{\underset{\text{D}}{\underset{\wr}{\underset{\text{A}}{24}}}}\quad\underset{\text{選 1}}{\underset{\text{D}}{\underset{\wr}{\underset{\text{A}}{25}}}}\quad\Rightarrow 4\times 4=16$

$\Rightarrow 3\times 16=48$

全部猜法$=4^5=1024$

連續三格的答案不要出現 BAD 之猜法$=1024-48=976$

22. 港口 A　$\Rightarrow -1\leqq\sin(\frac{2\pi}{11}t)\leqq 1\quad\Rightarrow -4\leqq 4\cdot\sin(\frac{2\pi}{11}t)\leqq 4$

$\Rightarrow 12\leqq 4\cdot\sin(\frac{2\pi}{11}t)+16\leqq 20\quad\Rightarrow 12\leqq y\leqq 20$

港口 B　$\Rightarrow -1\leqq\cos(\frac{2\pi}{13}t)\leqq 1\quad\Rightarrow 5\geqq -5\cos(\frac{2\pi}{13}t)\geqq -5$

$\Rightarrow 22\geqq -5\cos(\frac{2\pi}{13}t)+17\geqq 12\quad\Rightarrow 12\leqq y\leqq 22$

23. $y(t)=\dfrac{600000}{1+2\times 2.7^{-0.01(t-2022)}}$

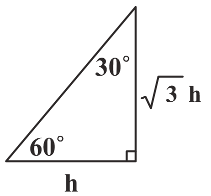
(B)$y(2022)=\dfrac{600000}{1+2\times 2.7^{-0.01\times 0}}=\dfrac{600000}{1+2\times 1}=200000$

24. 男廁　$\Rightarrow$ A , B , C , D

$\left.\begin{array}{l}\text{留1間}\quad\Rightarrow C_1^4=4\\ \text{留2間}\quad\Rightarrow C_2^4=6\end{array}\right\}$樣本空間$=4+6=10$

$P_{\text{剩 2 間男廁}}=\dfrac{6}{10}=\dfrac{3}{5}$

25. (1)



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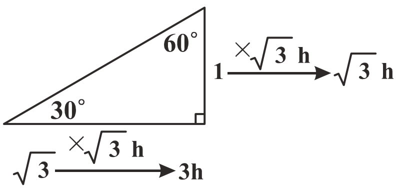

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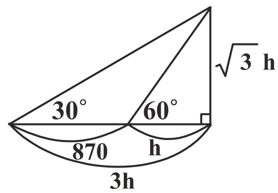

$870+h=3h$

$\therefore h=435$

涼亭的海拔高度$=3+\sqrt{3}\,h=3+435\sqrt{3}$