

# 112 學年度科技校院四年制與專科學校二年制

## 統一入學測驗公告答案

考科代碼：4-00-MC

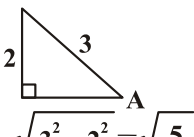
類 別：共同科目

考 科：數學(C)

| 題號 | 答案 | 題號 | 答案 | 題號 | 答案 | 題號 | 答案 | 題號 | 答案 | 題號 | 答案 |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 1  | A  | 11 | C  | 21 | B  | 31 |    | 41 |    | 51 |    |
| 2  | A  | 12 | D  | 22 | C  | 32 |    | 42 |    | 52 |    |
| 3  | B  | 13 | B  | 23 | D  | 33 |    | 43 |    | 53 |    |
| 4  | C  | 14 | C  | 24 | D  | 34 |    | 44 |    | 54 |    |
| 5  | B  | 15 | D  | 25 | B  | 35 |    | 45 |    | 55 |    |
| 6  | D  | 16 | A  | 26 |    | 36 |    | 46 |    | 56 |    |
| 7  | A  | 17 | D  | 27 |    | 37 |    | 47 |    | 57 |    |
| 8  | A  | 18 | B  | 28 |    | 38 |    | 48 |    | 58 |    |
| 9  | A  | 19 | A  | 29 |    | 39 |    | 49 |    | 59 |    |
| 10 | C  | 20 | B  | 30 |    | 40 |    | 50 |    | 60 |    |

1.  $4x^2 - 2x - 5 = 0$  二根積  $\alpha \beta = \frac{-5}{4}$

2. (A)  $\sqrt[3]{x^3} = x$  (對) ; (B)  $\sqrt{4+x^2} = 2+x \xrightarrow{\text{平方}} 4+x^2 \neq 4+4x+x^2$  (錯) ;  
 (C)  $\sqrt{x^2} = |x|$  (錯) ; (D)  $\sqrt[3]{8-x^3} = 2-x \xrightarrow{\text{三次方}} 8-x^3 \neq 8-12x+6x^2-x^3$  (錯)

3.   $\Rightarrow \tan A = \frac{2}{\sqrt{5}}$

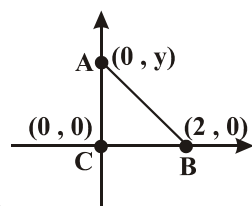
4.  $\overline{BC}$  中點為 E  $\Rightarrow E(1, 0)$

$\overline{AB}$  中點為 D  $\Rightarrow D(\frac{x+2}{2}, \frac{y}{2}) = (\frac{x}{2} + 1, \frac{y}{2})$

$\overline{AC}$  中點為 F  $\Rightarrow F(\frac{x}{2}, \frac{y}{2})$

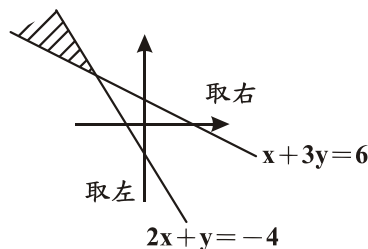
$\vec{DE} = (-\frac{x}{2}, -\frac{y}{2})$ ,  $\vec{DF} = (-1, 0)$

$\vec{DE} \cdot \vec{DF} = 0 \Rightarrow \frac{x}{2} + 0 = 0 \Rightarrow x = 0 \Rightarrow A(0, y)$

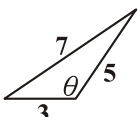


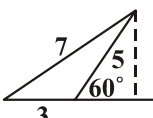
$\Rightarrow$  選(C)

5. 
$$\begin{cases} x+3y \geq 6 & \begin{array}{c|cc} x & 6 & 0 \\ y & 0 & 2 \end{array} \\ 2x+y \leq -4 & \begin{array}{c|cc} x & -2 & 0 \\ y & 0 & -4 \end{array} \end{cases}$$



$\Rightarrow$  第二象限

6.   $\cos \theta = \frac{3^2 + 5^2 - 7^2}{2 \times 3 \times 5} = -\frac{1}{2} \Rightarrow \theta = 120^\circ$

  $\text{高} = 5 \times \frac{\sqrt{3}}{2} = \frac{5}{2} \sqrt{3}$

$$7. C=AB=\begin{bmatrix}-1 & 1 & 0 \\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1 \\ 1 & 0 \\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1 \\ 0 & 1\end{bmatrix}$$

$$C^2=\begin{bmatrix}1 & -1 \\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & -1 \\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -2 \\ 0 & 1\end{bmatrix}=\begin{bmatrix}d_{11} & d_{12} \\ d_{21} & d_{22}\end{bmatrix}$$

$$d_{12}=-2$$

$$\begin{aligned}8. \frac{x^2y^4}{z^3} &= \frac{[\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})]^2 \times [2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})]^4}{[2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})]^3} \\&= \frac{[\sqrt{2}^2(\cos(\frac{\pi}{4}+\frac{\pi}{4})+i\sin(\frac{\pi}{4}+\frac{\pi}{4}))][2^4(\cos(\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{6})+i\sin(\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{6}))]}{2^3(\cos(\frac{\pi}{3}+\frac{\pi}{3}+\frac{\pi}{3})+i\sin(\frac{\pi}{3}+\frac{\pi}{3}+\frac{\pi}{3}))} \\&= \frac{[2(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})][16(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi)]}{8(\cos\pi+i\sin\pi)} \\&= 4[\cos(\frac{\pi}{2}+\frac{2}{3}\pi-\pi)+i\sin(\frac{\pi}{2}+\frac{2}{3}\pi-\pi)] \\&= 2^2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})\end{aligned}$$

$$9. \vec{AB}=(1, 2, 3)$$

$$\vec{AC}=(2, 3, 1)$$

$$\Rightarrow \vec{AB} \times \vec{AC}=(-7, 5, -1) \Rightarrow \text{則取 } E_1 \text{ 的法向量 } \vec{n}_1=(7, -5, 1)$$

$$\text{又 } E_2 \text{ 的法向量 } \vec{n}_2=(1, k, -2) \Rightarrow \vec{n}_1 \perp \vec{n}_2 \Rightarrow \vec{n}_1 \cdot \vec{n}_2=0$$

$$\Rightarrow 7-5k-2=0 \Rightarrow k=1$$

$$10. \text{ 將 } x=1, y=1, z=-1 \text{ 代入方程組 } \begin{cases} a+b-c=-2 \cdots \cdots (1) \\ b+c-a=-4 \cdots \cdots (2) \\ c+a-b=6 \cdots \cdots (3) \end{cases}$$

$$(1)+(2)+(3): a+b+c=0 \cdots \cdots (4)$$

$$\frac{(4)-(1)}{2}: c=1, \frac{(4)-(2)}{2}: a=2, \frac{(4)-(3)}{2}: b=-3 \Rightarrow ac=2$$

$$11. \text{ 取 } \frac{-2+6}{2}=2, x \leq -2 \text{ 或 } x \geq 6 \xrightarrow{-2} x-2 \leq -4 \text{ or } x-2 \geq 4$$

$$\Rightarrow |x-2| \geq 4 \xrightarrow{\times 3} |3x-6| \geq 12$$

$$\Rightarrow |-3x+6| \geq 12 \text{ 與 } |Ax+6| \geq B \text{ 相同 } \Rightarrow \begin{cases} A=-3 \\ B=12 \end{cases} \Rightarrow 2A+B=6$$

12. 只有三根，表  $b+2i$  與  $-1+ai$  為共軛複數

$$\Rightarrow \begin{cases} b=-1 \\ a=-2 \end{cases} \Rightarrow \text{三根為 } -2, -1+2i, -1-2i$$

$$x = -1 \pm 2i \Rightarrow x+1 = \pm 2i \xrightarrow{\text{平方}} x^2 + 2x + 1 = -4$$

$$\Rightarrow x^2 + 2x + 5 = 0 \quad \therefore f(x) = (x+2)(x^2 + 2x + 5)$$

$$\Rightarrow f(x) = x^3 + 4x^2 + 9x + 10$$

13. 2018 年~2023 年  $\Rightarrow 5$  年，若每年年利率  $r \Rightarrow 50(1+r)^5 = 60$

14.  $\frac{2000}{250} = 8$  小時  $\Rightarrow 8 - 0.5 - 6.5 = 1$  小時

兔子只剩 1 小時可追

$$\text{又 } s(t) = \int v(t)dt = \int (27t^2 + 52t + 1262)dt = 9t^3 + 26t^2 + 1262t + c \quad \text{又 } s(0) = c = 600$$

$$\Rightarrow s(t) = 9t^3 + 26t^2 + 1262t + 600 \Rightarrow s(1) = 9 + 26 + 1262 + 600 = 1897$$

$$2000 - 1897 = 103$$

15. 原式 =  $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 8n - 3}^2 - \sqrt{n^2 + 2n + 5}^2}{\sqrt{n^2 + 8n - 3} + \sqrt{n^2 + 2n + 5}} = \lim_{n \rightarrow \infty} \frac{6n - 8}{\sqrt{n^2 + 8n - 3} + \sqrt{n^2 + 2n + 5}} \quad \frac{\text{上下}}{\text{除}n}$

$$\lim_{n \rightarrow \infty} \frac{6 - \frac{8}{n}}{\sqrt{1 + \frac{8}{n} - \frac{3}{n^2}} + \sqrt{1 + \frac{2}{n} + \frac{5}{n^2}}} = \frac{6}{\sqrt{1} + \sqrt{1}} = 3$$

16. 設圓方程式  $x^2 + y^2 + dx + ey + f = 0$

$$A(1, 2) \text{ 代入 } 1 + 4 + d + 2e + f = 0 \cdots \cdots (1)$$

$$B(2, -3) \text{ 代入 } 4 + 9 + 2d - 3e + f = 0 \cdots \cdots (2)$$

$$C(2, 7) \text{ 代入 } 4 + 49 + 2d + 7e + f = 0 \cdots \cdots (3)$$

$$(3) - (2) : 40 + 10e = 0 \Rightarrow e = -4$$

$$\text{代入}(1) \Rightarrow 5 + d - 8 + f = 0 \Rightarrow d = -28$$

$$(2) \Rightarrow 13 + 2d + 12 + f = 0 \Rightarrow f = 31$$

$$\Rightarrow x^2 + y^2 - 28x - 4y + 31 = 0 \quad D(a, -10) \text{ 代入 } a^2 + 100 - 28a + 40 + 31 = 0$$

$$\Rightarrow a^2 - 28a + 171 = 0 \Rightarrow (a-19)(a-9) = 0 \Rightarrow a = 19 \text{ 或 } 9$$

17. 小明： $30000 + x \cdot 3000 = 80000 \Rightarrow 3000x = 50000 \Rightarrow x \doteq 16.67$

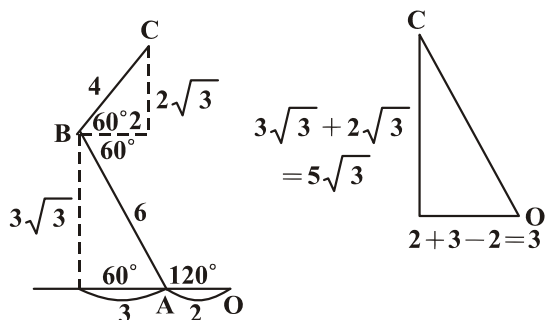
$$\text{小亮：} 30000 \times (1 + \frac{3}{100})^y = 80000 \Rightarrow 1.03^y = \frac{8}{3} \xrightarrow[\log]{\text{取}} \log 1.03^y = \log \frac{8}{3}$$

$$\Rightarrow y \times \log 1.03 = \log 2^3 - \log 3 \Rightarrow 0.0128y = 0.903 - 0.4771 = 0.4259$$

$$\Rightarrow y = \frac{0.4259}{0.0128} \doteq 33.27$$

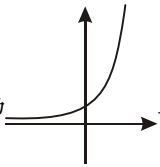
$$x - y \doteq 16.67 - 33.27 \doteq -17$$

18.



$$\Rightarrow \overline{OC} = \sqrt{3^2 + (5\sqrt{3})^2} = \sqrt{84} = 2\sqrt{21}$$

19.  $\log_{10}E = 11.8 + 1.5M \Rightarrow E = 10^{11.8+1.5M} = 10^{11.8} \times (10^{1.5})^M$ ,

若  $y = a^x$ ,  $a > 1 \Rightarrow$  圖形為  且真數 E 必大於 0

20. 主菜紅肉，湯白肉： $5 \times 2 \times 2 \times 4 = 80$

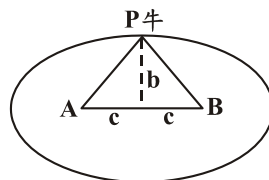
主菜白肉，湯紅肉： $5 \times 2 \times 2 \times 4 = 80$

$$\Rightarrow 80 + 80 = 160$$

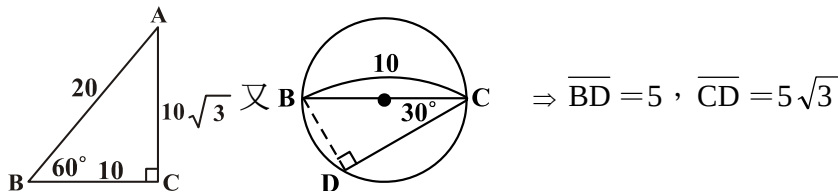
21. 依橢圓定義  $\overline{PA} + \overline{PB} = 8 = 2a \Rightarrow a = 4$

又  $2c = 4 \Rightarrow c = 2$ ,  $a^2 = b^2 + c^2 \Rightarrow 16 = b^2 + 4 \Rightarrow b = 2\sqrt{3}$

而牛距離  $\overleftrightarrow{AB}$  最遠為  $b = 2\sqrt{3}$



22.



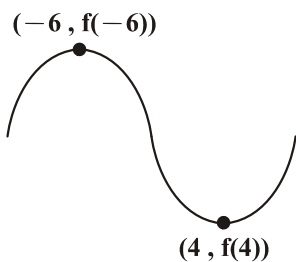
$$\Rightarrow \overline{AD} = \sqrt{(10\sqrt{3})^2 + (5\sqrt{3})^2} = \sqrt{375} = 5\sqrt{15}$$

23.  $f'(x) = \frac{1}{2\sqrt{x}} - 1$  過  $(1, 0)$  的切線  $m = f'(1) = \frac{1}{2\sqrt{1}} - 1 = \frac{-1}{2}$

兩切線垂直，則過  $g(x)$  以  $(1, 0)$  為切點的切線斜率  $m = g'(1) = 2$  ( $\because m \times m' = -1$ )

$$\left. \begin{array}{l} g'(x) = 3ax^2 + b \Rightarrow g'(1) = 3a + b = 2 \\ \text{又切點}(1, 0) \Rightarrow g(1) = a + b = 0 \end{array} \right\} \Rightarrow \begin{array}{l} a = 1 \\ b = -1 \end{array} \Rightarrow ab = -1$$

24.  $f'(x) = 3x^2 + 6x - 72 = 3(x^2 + 2x - 24) = 3(x+6)(x-4) = 0$ ,  $x$  在  $-6, 4$  產生極值  
 $x = -6$  時，相對極大值  $f(-6)$  }  $\rightarrow$  選(D)  
 $x = 4$  時，相對極小值  $f(4)$  }



25.  $f(x) = \sin x$  的週期為  $2\pi$   
 $f(x) = 40\sin(ax + \frac{\pi}{2}) + 70$   
 週期為  $\frac{2\pi}{a} = 4 \Rightarrow a = \frac{\pi}{2}$